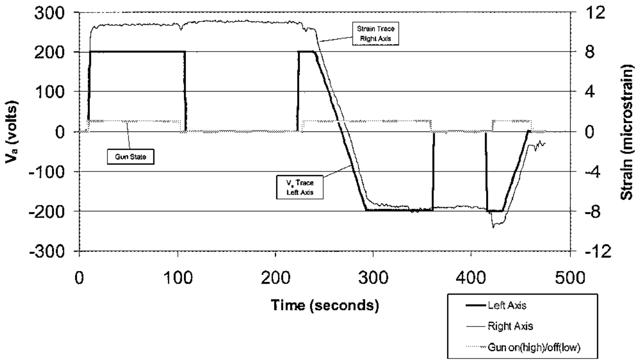

Fig. 11 Electron gun state V_a and resultant strain as a function of time. Note that the piezoelectric strain remains in the material for significant periods of time even with the gun deactivated and V_a is removed.

100 s. At 215 s, V_a is stepped back to the 200-V level, and the gun is turned on immediately thereafter. With the circuit reestablished, V_a is then ramped slowly to -200 V to generate a strain of $-8\ \mu\varepsilon$. V_a and the gun current are then deactivated again to demonstrate the permanence of the shape changes. The current is then reestablished by returning to the original V_a, and gun current levels and the piezoelectric strain is ramped back toward zero. Clearly evident in this simple test are the controllability, reversibility, and short-term stability of piezoelectric strains stimulated by electron gun charge and V_a inputs.

Conclusions

Strain control of a piezoelectric plate using an electron gun to deposit charge on the bare face of the material and a single distributed electrode on the opposite side was demonstrated to be stable, controllable, and repeatable. The responses observed while using the 300- and 400-eV electron beams are similar in linearity and hysteresis to piezoelectric responses using the conventional paired electrode method. It was also demonstrated in tests using a 400-eV beam that charge input can be applied to the material and that the resultant strains will remain for a significant period of time after the removal of the electron beam. Clearly nonlinear behavior due to unstable points in the secondary emission curve were evident in the 250-eV beam tests, indicating that higher beam energies are more desirable for stable operation.

The electron beam shape control method appears to hold potential for applications requiring high-resolution shape control of a large structure with extremely high resolution. Given that electron guns require vacuum to operate effectively, space is obviously one place to look for potential applications. Two specific possible uses involve the gathering of light. First, solar thermal propulsion units could utilize the shape correction possibilities of an electron gun controlled mirror to gather and focus the desired solar radiation. Second, orbital telescopes used in cosmology or Earth observations could take advantage of the ability to control very large surface areas to optical tolerances. The advantage of a large telescope is that the more light that is gathered then the better the telescope performs. The manufacture of extremely large primary and secondary reflectors may become much easier and cheaper if a practical method of on-orbit shape correction can be developed and implemented. Based on the research performed to date, electron gun shape control of piezoelectric materials may provide one possible avenue for performing on-orbit shape tuning of extremely large optical components.

Acknowledgments

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Improved Calculation of Eigenvalue Variation in Dynamic System

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Introduction

THE prediction of the modal change following a modification in a dynamic system is useful in many problems of engineering practice. This type of study was addressed as early as in Rayleigh's book.[1] It is noted that the formulas of either perturbation,[2,3] sensitivity,[4] or variation[5] are essentially of a similar form and hence they are considered as instances of the classic method. Normally, to improve the accuracy of an analysis, two ways are followed: the addition of higher variations and the utilization of an iterative process.[6,7] The step-by-step procedure based on the division of a large modification into small ones is an alternative.[7] The use of the Rayleigh quotient is a good way of approximation because it involves a second-order error.[2] Bickford[8] proposed a modified first variation, which can make up the Rayleigh quotient. The incorporation of the Rayleigh quotient into a perturbation analysis can also improve the convergence of the eigenvalues as well as the eigenvectors.[9,10]

A procedure for the improved calculation of eigenvalue variations is presented in this Note.

Classic Method

For a dynamic system in the original state, the eigenvalue equation is in a form of

$$\left(\boldsymbol{A}_0 - \lambda_{0_i}\boldsymbol{B}_0\right)\boldsymbol{x}_{0_i} = \boldsymbol{0} \tag{1}$$

where $\boldsymbol{A}_0$ and $\boldsymbol{B}_0$ are real symmetrical matrices and λ_{0_i} and $\boldsymbol{x}_{0_i}$ are ith eigenvalue and eigenvector, respectively. The orthonormality relationships can be defined as

$$\boldsymbol{x}_{0_i}^T\boldsymbol{A}_0\boldsymbol{x}_{0_j} = \delta_{ij}\lambda_{0_i} \tag{2}$$

$$\boldsymbol{x}_{0_i}^T\boldsymbol{B}_0\boldsymbol{x}_{0_j} = \delta_{ij} \tag{3}$$

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where the Kronecker delta $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$. The modified matrices are assumed to be

$$A = A_0 + \delta A \tag{4}$$

$$B = B_0 + \delta B \tag{5}$$

where δA and δB are the first variations of the matrices. The eigenvalue equation and the normality of the modified system are then

$$(A - \lambda_i B)x_i = 0 \tag{6}$$

$$x_i^T B x_i = 1 \tag{7}$$

The eigensolution is given in the truncated series, expressed as

$$\lambda_i = \lambda_{0_i} + \delta\lambda_i + \delta^2\lambda_i + \mathcal{O}(\varepsilon^3) \tag{8}$$

$$x_i = x_{0_i} + \delta x_i + \delta^2 x_i + \mathcal{O}(\varepsilon^3) \tag{9}$$

where $\delta\lambda_i$, $\delta^2\lambda_i$, δx_i, and $\delta^2 x_i$ are the first and second variations. By letting $\delta A = \varepsilon A^{(1)}$ and $\delta B = \varepsilon B^{(1)}$, where ε is a small parameter, and using the perturbation theory,[2,3] the first and second variations can be obtained as

$$\delta\lambda_i = x_{0_i}^T\left(\delta A - \lambda_{0_i}\delta B\right)x_{0_i} \tag{10}$$

$$\delta x_i = -\frac{1}{2}x_{0_i}^T\delta B x_{0_i} x_{0_i} + \sum_{\substack{j=1\\ j\neq i}}^{n}\frac{x_{0_j}^T\left(\delta A - \lambda_{0_i}\delta B\right)x_{0_i}}{\lambda_{0_i} - \lambda_{0_j}}x_{0_j} \tag{11}$$

$$\delta^2\lambda_i = x_{0_i}^T\left(\delta A\delta x_i - \lambda_{0_i}\delta B\delta x_i - \delta\lambda_i B_0\delta x_i - \delta\lambda_i\delta B x_{0_i}\right) \tag{12}$$

$$\delta^2 x_i = -\left(\frac{1}{2}\delta x_i^T B_0 + x_{0_i}^T\delta B\right)\delta x_i x_{0_i} + \sum_{\substack{j=1\\ j\neq i}}^{n}\frac{x_{0_j}^T\left(\delta A\delta x_i - \lambda_{0_i}\delta B\delta x_i - \delta\lambda_i B_0\delta x_i - \delta\lambda_i\delta B x_{0_i}\right)}{\lambda_{0_i} - \lambda_{0_j}}x_{0_j} \tag{13}$$

Taking the first variation directly to Eq. (1) can also result in the first variations.[5]

Improved Calculation

It is noted that the variations of eigenvalue and eigenvector in Eq. (6) come from the variations of the variations δA and δB. The first variation of eigenvalue $\delta\lambda_i$ is a function of the variables δA and δB, and so is the first variation of eigenvector δx_i. Therefore, $\delta\lambda_i$ and δx_i, which are independent from each other, should be implied functions in Eq. (6), expressed as

$$\delta\lambda_i = f_1(\delta A, \delta B) \tag{14}$$

$$\delta x_i = f_2(\delta A, \delta B) \tag{15}$$

For this reason, $\delta\lambda_i$ is a function in

$$\left[(A_0 + \delta A) - \left(\lambda_{0_i} + \delta\lambda_i\right)(B_0 + \delta B)\right]x_{0_i} = 0 \tag{16}$$

The extension of the equation is

$$\left(A_0 - \lambda_{0_i}B_0\right)x_{0_i} + \left(\delta A - \lambda_{0_i}\delta B\right)x_{0_i} - \delta\lambda_i(B_0 + \delta B)x_{0_i} = 0 \tag{17}$$

Premultiplying Eq. (17) by $x_{0_i}^T$ and applying the normality relationships from Eqs. (2) and (3), the first variation of eigenvalue is obtained as

$$\delta\lambda_i = \frac{x_{0_i}^T\left(\delta A - \lambda_{0_i}\delta B\right)x_{0_i}}{1 + x_{0_i}^T\delta B x_{0_i}} \tag{18}$$

Taking δx_i as a function in Eqs. (6) and (7) and keeping the terms of linear variation can yield

$$\left(\delta A - \lambda_{0_i}\delta B\right)x_{0_i} + \left(A_0 - \lambda_{0_i}B_0\right)\delta x_i = 0 \tag{19}$$

$$x_{0_i}^T B_0\delta x_i + x_{0_i}^T\delta B x_{0_i} + \delta x_i^T B_0 x_{0_i} = 0 \tag{20}$$

Inserting δx_i, which is expanded as

$$\delta x_i = \sum_{k=1}^{n}\beta_{ik}x_{0_k} \tag{21}$$

into Eq. (19) and premultiplying the equation by $x_{0_j}^T$, for $j \neq i$, the coefficient β_{ij} is obtained as

$$\beta_{ij} = \frac{x_{0_j}^T\left(\delta A - \lambda_{0_i}\delta B\right)x_{0_i}}{\lambda_{0_i} - \lambda_{0_j}}, \qquad j \neq i \tag{22}$$

Substituting Eq. (21) into Eq. (20) can yield the coefficient β_{ii}:

$$\beta_{ii} = -\tfrac{1}{2}x_{0_i}^T\delta B x_{0_i} \tag{23}$$

Inserting Eqs. (22) and (23) into Eq. (21), the first variation of eigenvector, as in Eq. (11), is obtained. The variation of eigenvector is a solution of the modal superposition. Other techniques can be considered.[6]

The second variation of eigenvalue $\delta^2\lambda_i$, as a function of first variations δA, δB, $\delta\lambda_i$, and δx_i, can be expressed in

$$\left[A_0 + \delta A - \left(\lambda_{0_i} + \delta\lambda_i + \delta^2\lambda_i\right)(B_0 + \delta B)\right]\left(x_{0_i} + \delta x_i\right) = 0 \tag{24}$$

From this equation, the second variation of eigenvalue can be obtained as

$$\delta^2\lambda_i = \frac{x_{0_i}^T\left(\delta A - \lambda_{0_i}\delta B - \delta\lambda_i B_0 - \delta\lambda_i\delta B\right)\delta x_i}{x_{0_i}^T(B_0 + \delta B)\left(x_{0_i} + \delta x_i\right)} \tag{25}$$

In the same way, the second variation $\delta^2 x_i$, as in Eq. (13), can be obtained.

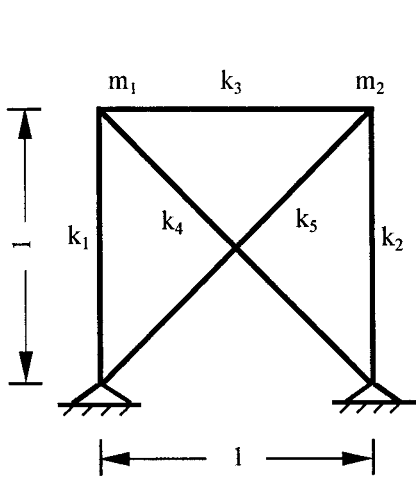

Fig. 1 Plane truss.

Comparison and Discussion

The improved calculation is different from the classic method in the eigenvalue variations. The second variations of eigenvector appear to be the same for both methods, but the values $\delta\lambda_i$ are different. Though there is one more term $\boldsymbol{x}_{0_i}^T \delta\boldsymbol{B}\boldsymbol{x}_{0_i}$ in the denominator of Eq. (18), no extra matrix operation is required, as it must be computed in Eq. (10). Similarly, matrix operations in Eq. (25) are the same as in Eq. (12).

The first variation of the classic method is linear to $\delta\boldsymbol{A}$ and $\delta\boldsymbol{B}$. However, the first variation in the improved calculation, which contributes an eigenvalue approximation exactly in the form of the Rayleigh quotient, is no longer linear to $\delta\boldsymbol{B}$. If $\delta\boldsymbol{B} = 0$, both Eqs. (10) and (18) become

$$\delta\lambda_i = \boldsymbol{x}_{0_i}^T \delta\boldsymbol{A}\boldsymbol{x}_{0_i} \tag{26}$$

Therefore, the variations are different from the classic method only when there is $\delta\boldsymbol{B}$.

Other approaches can be used to derive the first variation of eigenvalue. Inserting $\boldsymbol{x}_{0_i}$ into the Rayleigh quotient can give a change of eigenvalue as the variation in Eq. (18). Bickford[8] obtained the first variation by adding the term $\boldsymbol{x}_{0_i}^T \boldsymbol{B}\boldsymbol{x}_{0_i}$ to the denominator of Eq. (10); $\boldsymbol{x}_{0_i}^T \boldsymbol{B}\boldsymbol{x}_{0_i}$ was also added to Eq. (11) for the eigenvector variation,

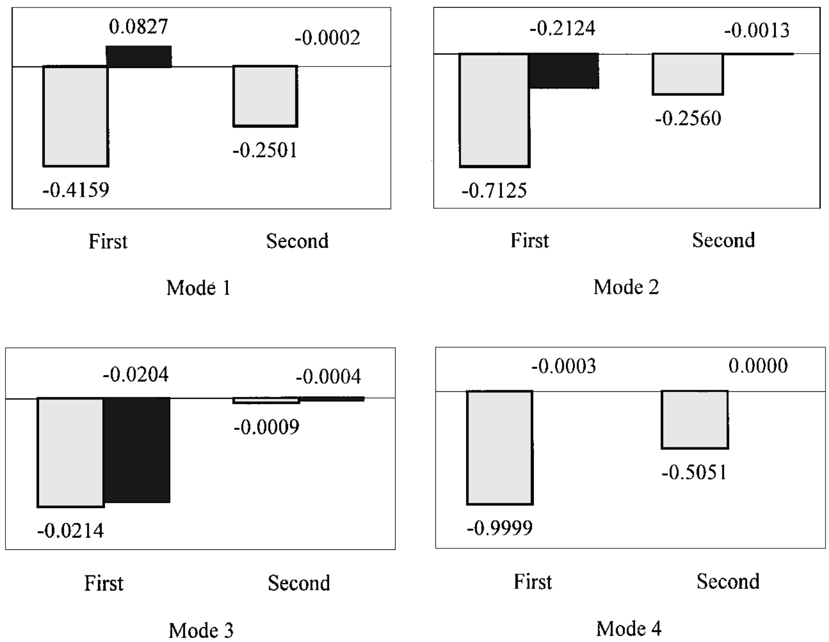

Fig. 2 Comparison of relative errors of first and second eigenvalue variations from the classic method (□) and the improved calculation (■) (percent).

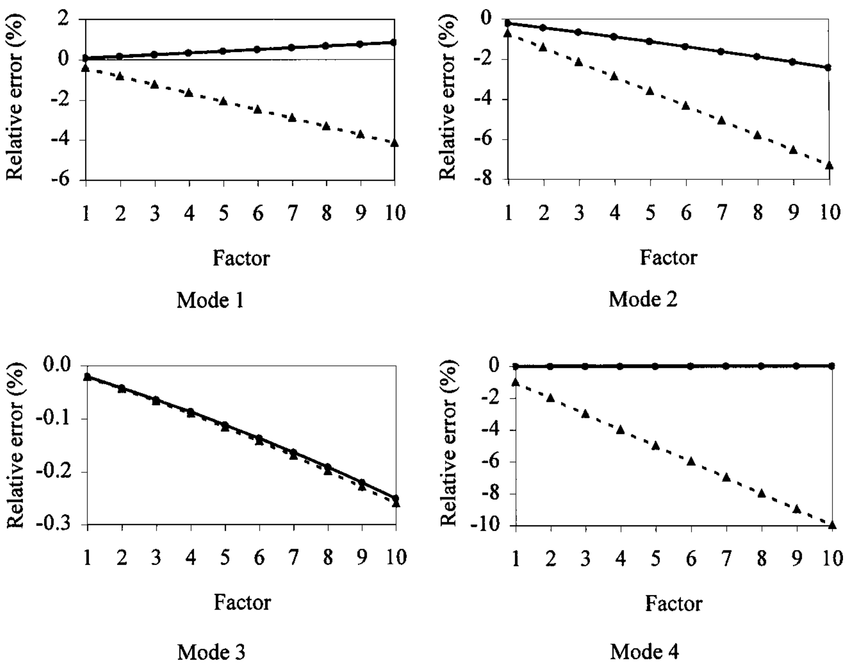

Fig. 3 Comparison of relative errors of first eigenvalue variations from the classic method (▲) and the improved calculation (●) vs multiplying factor (percent).

Table 1 Comparison of first variations and eigenvalues from the classic method and the improved calculation

Mode	Parameter	Exact	Classic	Improved
1	$\delta\lambda_1$	0.001694	0.001687	0.001696
	λ_1	0.330121	0.330114	0.330122
2	$\delta\lambda_2$	0.031665	0.031439	0.031598
	λ_2	2.343013	2.342787	2.342946
3	$\delta\lambda_3$	0.010113	0.010111	0.010111
	λ_3	4.901943	4.901941	4.901941
4	$\delta\lambda_4$	0.069529	0.068834	0.069529
	λ_4	5.952137	5.951442	5.952137

but the accuracy is not as good as that of the eigenvalue.[8] The first variation was also obtained by Lü et al.[7]

Numerical Example

A plane truss (Fig. 1) is chosen as the example, with the sectional stiffness for five bars and the lumped nodal masses taken as (5.5, 4.5, 1, 1, 1) and (1, 1). The variations of stiffness and masses are assumed to be (0.01, 0.01, 0.01, 0, 0) and (−0.01, 0). The first variations and the eigenvalues obtained by the classic method and the improved calculation are given in Table 1. The relative errors of the variations are shown in Fig. 2. The result of the improved calculation is in the very good agreement with the exact one. Apart from the third mode, the first variations given by the improved calculation are even better than those with the second variations by the classic method.

A large modification of the truss is considered by multiplying $\delta\boldsymbol{A}$ and $\delta\boldsymbol{B}$ with a factor from 1 through 10. The relative errors of the first variations are shown in Fig. 3. The errors in the first mode increase from 0.0827 to 0.8410%, whereas those of the classic method change from −0.4159 to −4.1834%. The fourth mode shows the biggest difference, as the largest error of the improved calculation is −0.0023% but −9.9982% for the classic method.

Conclusions

An improved calculation of eigenvalue variations has been presented. The eigenvalue approximation from the first variation is a Rayleigh quotient. The variations are different from those in the classic method when there is a variation of $\boldsymbol{B}$, which is the inertia-related matrix in a structural dynamics. Although there are more terms in the variations of eigenvalues, no extra matrix operation is required. The given example confirms that the accuracy of the eigenvalue variations is much better than that obtained with the classic method.

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Numerical Presentation of Eigenvalue Curve Veering and Mode Localization in Coupled Pendulums

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Introduction

THE eigenvalue curve veering and mode localization are the phenomena to show a rapid modal change in dynamic systems.[1–9] It was demonstrated that a mistuned length to a pendulum in weakly coupled pendulums can result in the occurrence of the phenomena.[6,8] This small irregularity is a disorder to the pendulums with a nearly periodic feature. Although their occurrence and orientation have been recognized in the published references, a standard to identify the phenomena must be further studied. This Note provides an attempt to use the derivatives of eigenvalue and eigenvector as the numerical presentation of the phenomena in the weakly coupled pendulums.

Curve Veering and Mode Localization

Two weakly coupled pendulums are shown in Fig. 1. The length of the left pendulum is given a disorder factor γ. The eigenvalue equation of the pendulums is such that

$$\left(\begin{bmatrix} 1/(1+\gamma)+R & -R/(1+\gamma) \\ -R-R\gamma & 1+R \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \begin{Bmatrix} \varphi_1 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{1}$$

where $R = kL/mg$, $\lambda = L\omega^2/g$, and ω is the angular frequency. The eigenvector is normalized as

$$(\varphi_1)^2 + (\varphi_2)^2 = 1 \tag{2}$$

Two pairs of the eigenvalues and eigenvectors can be directly solved and are given as

$$\lambda_{1,2} = \frac{2 + 2R + \gamma + 2R\gamma \mp T}{2 + 2\gamma} \tag{3}$$

$$\{\varphi_1 \quad \varphi_2\}_i = \frac{\{1 \quad S_i\}}{U}, \qquad i = 1, 2 \tag{4}$$

where $T = \sqrt{[\gamma^2 + 4R^2(1+\gamma)^2]}$, $S_i = [1 + (R - \lambda_i)(1+\gamma)]/R$, and $U = \sqrt{(1 + S_i^2)}$.

It was found that the values of γ and k determine the occurrence of the curve veering and mode localization as well as the violent degree of the phenomena.[6,8] As a condition, k must be small so that two eigenvalues are close. Furthermore, the phenomena only occur for a small γ. Giving $k = 0.01$, the eigenvalues and eigenvectors in Fig. 2 are showing the curve veering and mode localization at $\gamma \approx 0$. The existence of the phenomena prevents the standard perturbation analysis to give a precise evaluation of the modal changes. A modified

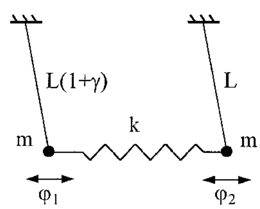

Fig. 1 Weakly coupled pendulums.

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